An Introduction to Utility Elicitation

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Overview

Loss aversion

Do you prefer:

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or

0$ with probability 0.2 and 4.000$ with probability 0.8?

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Notation

We will denote the lottery:
0$ with probability 0.2 and 4.000$ with probability 0.8 by $[0.2, 0; 0.8, 4.000]$

Cummulative Prospect theory

Captures key features of human behaviour:

- Loss aversion
- Probability weighting.

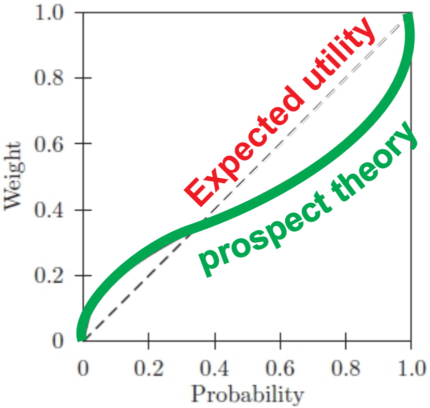

Figure 1: A probability weighting function $w(p)$.

Calculating expected utility

Under expected utility theory the lottery $[p, x; 1-p, z] \sim y$ gives utility:

$$p \cdot u(x) + (1-p) \cdot u(y).$$

Under cummulative prospect theory the lottery $[p, x; 1-p, z] \sim y$ gives utility:

$$w(p)u(x) + (1 - w(p))u(y).$$

Definitions

A decision scenario

- Decisions D are probability distributions over a finite set of outcomes $X = [x_0, \ldots, x_n]$.
- The utility $u : X \to [0, 1]$ of the user is a continous, increasing, private function with $u(x_0) = 0$ and $u(x_n) = 1$.
- The user has a continous, increasing probability weighting function w with $w(0) = 0$ and $w(1) = 1$.

CE and PE Methods

Both based in the indifference: $[p, x; 1-p, z] \sim y$ where $(x < y < z)$

CE: the analyst asks the user to give the outcome z

PE: the analyst asks the user to give the probability p

$[p, x; 1-p, z] \sim y$ gives $w(p)u(x) + (1-w(p))u(z) = u(y)$

Gamble Tradeoff (Wakker and Deneffe 1996)

The analyst asks the client
for X such that $[p, X; 1-p, r] \sim [p, x; 1-p, R]$
and Y such that $[p, Y; 1-p, r] \sim [p, y; 1-p, R]$
where $R > r > X > x$ and $R > r > Y > y$

Properties:

$$u(X) - u(x) = u(Y) - u(y) = \frac{1 - w(p)}{w(p)}(u(R) - u(r))$$

Taking $X = y = x_1, x = x_0$ and $Y = x_2$ we get $u(x_2) = 2u(x_1)$ So we can get a grid of n points $x_1, \ldots, x_n$ s.t. $u(x_i) = iu(x_1)$ where $u(x_1) = \frac{1-w(p)}{w(p)}(u(R) - u(r))$

Gamble Tradeoff (Wakker and Deneffe 1996)

Remarks:

- The weights w cancel so it applies to cummulative prospect theory and RDU
- The distance between any two points is: $u(x_1) = \frac{1-w(p)}{w(p)}(u(R) - u(r))$
- R and r are artificially big, bigger than any alternative

Larson-Hines

Main idea:

For any two outcomes s, t find their midpoint i.e. an outcome v s.t. $u(v) = \frac{u(s)+u(t)}{2}$ where $s < v < t$

The indifference $[1-p, s; p, t] \sim [p, s; 1-p, v]$ is equivalent to

$$u(v) - u(s) = \frac{w(p)}{w(1-p)}(u(t) - u(s)) \qquad (1)$$

if p is s.t. $\frac{w(p)}{w(1-p)} = \frac{1}{2}$ then (1) gives $u(v) = \frac{u(s)+u(t)}{2}$.

if we take $s = x_0, v = x_n$ then (1) becomes
$\frac{w(p)}{w(1-p)} = \frac{1}{u(t)} = \ldots = \frac{1}{2}$.
It just remains to find z s.t. $u(z) = 2$

Larson-Hines

Finding z with $u(z) = 2$

The idea is to measure the distance x_0 to x_1 wich is 1 using a probability p and to then dublicate this to find z with $u(z) = 2$.

Remembering the Wakker Method if we had p and z s.t.
$[p, x_0; 1 - p, r] \sim [p, x_n; 1 - p, R]$ and
$[p, x_n; 1 - p, r] \sim [p, z; 1 - p, R]$ then it would be $u(z) = 2u(x_n)$ and since since $u(x_n) = 1$ it would be $u(z) = 2$.
Well we just ask the user to give us the p and z that satisfy the previous two equivalences.

Hines-Larson made simpler

Main idea:

For any two outcomes s, t find their midpoint i.e. an outcome v s.t. $u(v) = \frac{u(s) + u(t)}{2}$ where $s < v < t$

The indifference $[p, s; 1 - p, t] \sim v$ is equivalent to

$$u(v) = w(p)u(s) + (1 - w(p))u(t) \tag{2}$$

if p is s.t. $w(p) = 1 - w(p) = \frac{1}{2}$ then (2) gives $u(v) = \frac{u(s)+u(t)}{2}$.

if we take $s = x_0, v = x_1$ then (2) becomes $u(t)(1 - w(p)) = 1$.
It just remains to find t s.t. $u(t) = 2$

Midweight method (elicits the weights w) [Van Kuilen Wakker 2009]

for every two probabilities $f < g$ we can find the probability e s.t. $w(e) = \frac{w(f)+w(g)}{2}$

Step 1: Use the Wakker method to elicit only two outcomes y_1, y_2 s.t. $u(y_2) = 2u(y_1)$

Step 2: Ask the user for the probability d s.t.

$$[b, x_0; c, y_1; a, y_2] \sim [b + (c - d), x_0; a + d, y_2]$$

where $a + b + c = 1$ and $0 < d < c$ this equivalence gives $w(d + a) = \frac{w(a)+w(c+a)}{2}$